Extending RCA algorithm to consider ternary relations

Nicolas Leutwyler[*†‡], Mario Lezoche[*†], Hervé Panetto[*†] and DiegoTorres[‡]
[*]Université de Lorraine, Nancy 5400, France
[†]Centre de Recherche en Automatique de Nancy,
Vandoeuvre-lès-Nancy 54506, France
[‡]LIFIA, CICPBA-Facultad de Informática, UNLP,
La Plata 1900, Argentina

***Abstract*—Relational Concept Analysis (RCA) is a multi-relational data mining method that aims to extract knowledge from multiple formal contexts (i.e., objects, attributes, and a binary relation between them) and the relations between them. One of the problems RCA has is the lack of the possibility of extracting knowledge directly from data that is represented with ternary relations. While there are some existing solutions towards this problem, either they require complex preprocessing of the input data, or they lose some capabilities of RCA such as the different meanings of the relations between concepts ($\exists, \forall$, etc). In this work, we present an intuitive extension to RCA to be able to use it with data directly represented with ternary relations. As an example of its usage, we apply it to a dataset called Knomana which includes ternary relations.**

***Index Terms*—Knowledge representation, Formal Concept Analysis, Relational Concept Analysis, ternary relations.**

I. Introduction

Agriculture 4.0 [1], [2], [3] is a term that stands for the next big trends facing the industry, including a greater focus on precision agriculture, the internet of things (IoT) and the use of big data to drive greater business efficiencies in the face of rising populations and climate change. One of the trends present in Agriculture 4.0 is the need of reducing the dependency on applying water, fertilizers, and pesticides across entire fields, so that the efficiency is increased in terms of resource utilization. Instead, farmers should use the minimum quantities, or, when it is possible, even completely remove them from the supply chain. Additionally, The Green Revolution of the 20th century was characterized by a blind use of pesticides and chemical fertilizers, resulting in a loss of soil biodiversity and a rise in resistance against pathogens and pests [4]. Furthermore, the Green Deal in the EU encourages the search for solutions to go towards a sustainable usage of pesticides. In this regard, Knomana [5] is a work that gathered knowledge in many fields, including the protection of agricultural individuals by the usage of vegetal species instead of pesticides.

In order to provide good usage of the information in Knomana to the experts, it is important to retrieve insightful information from it [6]. Formal Concept Analysis (FCA) [7] appears to be a good fit for this task, due to its inherent qualities for structuring and classifying data through conceptual structures that provide a relevant support for data exploration. On the other hand, in the Knomana dataset, there is a part that links plants through a ternary relation consisting of an agricultural individual, a pest that attacks it, and a defense against the pest. For extracting knowledge from linked data, one of the most used approaches is to apply multi-relational data mining (MRDM) methods [8], [9]. One option is the MRDM FCA extension, called Relational Context Analysis (RCA), presented in [10], which, in short, aims to link the concepts extracted by FCA by using *binary* relations between potentially different types of objects. Transforming data represented with ternary relations into a traditional relational context family is not a trivial task, and it generally does not scale in the amount of data as it is shown in [11]. Instead of modelling the data onto a traditional relational context family, we can extend the algorithm so that it accepts ternary relations (i.e., relations represented by three objects that might be from different contexts), and with that create a new knowledge graph structure that can be efficiently searched through. In this paper, we explore such extension, and apply it to the Knomana dataset as an example of its usage. Moreover, the method is general, and it is not particularly bounded to the specific context of this dataset.

II. Related Work

Some FCA extensions already covered the lack of ternary relations in traditional RCA.

Firstly, Graph-FCA (G-FCA) is presented in [12], as a way to extend FCA to *knowledge graphs*. In the extension, graph entities are FCA objects, and graph relationships are FCA attributes. Consequently, the incidence relation consists of tuples of objects (with various arities) related to attributes, rather than single objects to attributes. Since an extensional representation is not a set of objects, but a set of tuples of objects, we can consider it a n-ary relation.

Secondly, the Triadic Concept Analysis (TCA) [13] which considers the case in which an *object* x has the *attribute* y under the *condition* z. In TCA, a triadic concept is a 4-tuple $K = (O, A, B, I)$, where O is the set of objects, A the set of attributes, B the set of conditions, and $I \subseteq O \times A \times B$ relating an object with an attribute under a certain condition.

Lastly, traditional Relational Concept Analysis can be also used as it is, but applying certain transformations and encodings to the source data [14]. Particularly, to represent the same information as in a ternary relation "an *organism* is protected by a *plant* against a certain *aggressor*", it is needed to have three object-attribute contexts (one for each type), and two object-object contexts being *protectedBy* and *treats*.

The extended RCA method we present in this paper aims to provide the ability to extract knowledge from data related with ternary relations, in a more obvious way e.g., maintaining all the previous concepts of traditional RCA, and without any pre-computation needed.

III. Preliminaries

A. Formal concept analysis

Formal concept analysis (FCA), defined in [15] - [16], is a method for extracting knowledge from a dataset called Formal Context, i.e., a table consisting of objects, attributes, and relations between objects and attributes. Formally, a formal context $\mathcal{K}$ is a triple (G, M, I), where G is a set of objects, M is a set of attributes, and I is an incidence matrix where if $I_{i,j}$ is 1, we say that the object o_i has the attribute a_j, otherwise o_i does not. Let $'$ be the derivation operation on a set of objects (dually, on a set of attributes) given by

$$X' = \{a \in A \mid \forall o \in X : I_{o,a}\}$$
$$Y' = \{o \in O \mid \forall a \in Y : I_{o,a}\}$$

A formal concept is a pair $C = (X, Y)$ where $X \subseteq O, Y \subseteq A$, $X' = Y$, and $Y' = X$. X is called the extent and Y the intent. For readability purposes, we note $C.E$ to the extent, and $C.I$ to the intent. The set of all the formal concepts and the relation of inclusion of extents form the so-called *concept lattice*, which is a partially ordered set, and is usually noted with the letter $\mathcal{L}$.

B. Relational concept analysis

While FCA aims to extract formal concepts from a formal context, and then extract association rules [17], one of its extensions, called Relational Concept Analysis (RCA), presented in [18], and [19], is used for a similar purpose but considering that data can be composed of several objects with different attributes each, and be related between them. This extension mainly provides a way of extracting relations between formal concepts (in the form of attributes), even if they are from different contexts (e.g., concepts might be related by some particular semantic such as "there exists an object of concept A related with an object of concept B").

The input of RCA is named Relational Context Family (RCF) and consists of a tuple (K, R) where K is a set of formal contexts and R is a set of binary relations between objects of the contexts, i.e., $(o_1, o_2) \in r$ iff o_1 is related to o_2 for $r \in R$. Let $K_j \in K$ be a formal context, we say that $\mathcal{L}_j$ is the concept lattice calculated from K_j, whereas C_i^j is the i-th concept of the lattice $\mathcal{L}_j$. Given two contexts $K_i, K_j \in K$ and a relation $r \in R$ such that $r \subseteq O_i \times O_j$ where O_x is the set of objects in the context K_x, the scaling process between K_i and K_j is defined by the algorithm 1, in which objects remain constant, for each concept $C \in \mathcal{L}_j$ an attribute with the name ρ r.name : C is added. Finally, for each object and for each new attribute added, the incidence between them is included according to the semantics of the relational scaling operator ρ, which is briefly explained in the Relational scaling section, and more broadly in [20].

Algorithm 1: Context scaling algorithm

Input: $K_i, \mathcal{L}_j, r, \rho$, a context, a lattice, a relation and a relational scaling operator respectively

Output: Scaled K_i

$O, A, I \leftarrow K_i$
$A^+ \leftarrow A \cup \{\rho\ r.\text{name} : C \mid C \in \mathcal{L}_j\}$
$I^+ \leftarrow I \cup \{(o, \rho\ r.\text{name} : C) \mid o \in O, C \in \mathcal{L}_j, \rho(o, r, C)\}$
return O, A^+, I^+

C. Relational scaling

Typically, the RCA main algorithm extends contexts within the relations, using a *relational scaling* operator. The most commonly used operators are $\exists$, $\forall$, and $\forall\exists$ to name a few. In this work, we will focus only in the $\exists$ operator, whose semantic is $\exists(o, r, C) = r(o) \cap C.E \neq \varnothing$ where $r(o) = \{x \mid (o, x) \in r\}$.

IV. The Knomana dataset

Knomana [5], coming from the composition of *knowledge* and *management*, is the name of a dataset initiated in 2015 by Pierre Martin, computer scientist in Cirad, and Pierre Silvie, entomologist in the Agricultural Research Institute for Development (IRAD). The method utilized for obtaining the data consisted in choosing a theme of interest, e.g., the protection of an agricultural crop using an extract of a plant species. Then, to identify a statement model of the targeted knowledge, formalized in the form of an ontology, to finally identify, in the literature, all the knowledge regarding the chosen model.

In Knomana, sub-datasets are stored in tables, in which each of them represent the knowledge base related to a specific interest theme. Chronologically, the first set of knowledge, called RAP (for *Pests of Poaceae* or *Ravageurs de Poaceae* in French), brought together the trophic chains of Lepidoptera species with stems or spikes (i.e., bio-aggressors) of African plants belonging to the families Poaceae, Cyperaceae and Typhaceae. The ontology associated with this set of knowledge connects the host plant, the pest of the host plant, the natural enemy of the pest, the territory where this trophic relationship was observed, and finally the bibliographic reference.

With RAP, the objective was to identify, for example, in a rice-growing lowland in Benin, the local non-cultivated plants (e.g., weeds) likely to harbour the pests of agricultural crops when a crop (e.g., rice, sorghum, maize) was absent. The intention was then to manage these non-cultivated areas, for example by cutting the plants, in order to limit the development of these bio-aggressors instead of spraying pesticides on the agricultural crop. In 2017, the knowledge base PPAf (for African Pesticide Plants) was started. It brings together knowledge on the uses of plants used in agriculture as an alternative to synthetic chemical pesticides. In 2018, the initial objective of this set was broadened within the framework of a project involving the collaboration of various entities:

Joseph Ki-Zerbo University (Burkina Faso), the IRAD from Cameroon, UMR ISEM and LIRMM (https://ur-aida.cirad.fr/nos-recherches/projets-et-expertises/knomana). In doing so, the uses of plants with a pesticide or antibiotic effect, whether they concern humans (human health and public health) or animal agricultural crops (animal health) and plants, have been identified. The health of the environment was approached through knowledge related to the unintended effects possibly reported on the natural enemies of pests.

A. PPAf Model

The part of the PPAf dataset in which we will focus describes a ternary relation between a *protected plant*, a *bio-aggressor*, and a *biopesticide*, see Figure 1. Although each of the three parts of the relation could be considered as different types (and hence, have their own contexts), in this case, we will consider them as the same, all compressed in the same context.

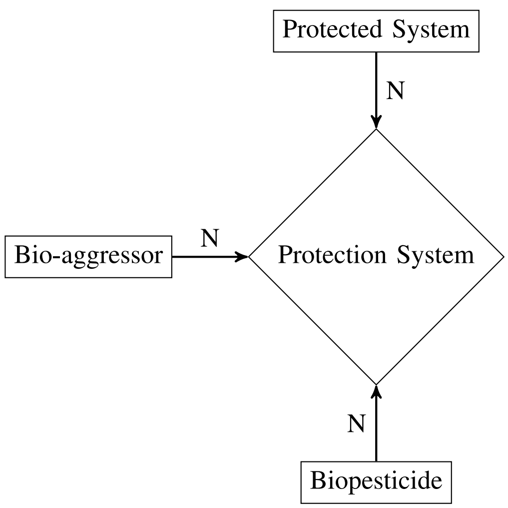

Fig. 1: Partial representation of the PPAf data model, showing the ternary relation

B. Extracting knowledge from PPAf

Using RCA in data represented with ternary relations requires some treatment, since the framework only accepts binary relations. It is important to state that ternary relations cannot be simply separated into three binary relations between their parts, i.e., $(\alpha, \beta) \wedge (\alpha, \gamma) \wedge (\beta, \gamma) \not\Rightarrow (\alpha, \beta, \gamma)$. For example, imagine four plants x, y, w, z, and two ternary relations (x, y, w), and (w, y, z), meaning that w protects x from the bio-aggressor y, and similarly that z protects w from the bio-aggressor y. If we had the binary relations (x, y), (w, y) to describe that y attacks both w and x, (y, w), (y, z) to describe that w and z protects against y, and (x, w), (x, z) to describe that w and z protect x, we would have to wonder for instance: *w protects x against whom?* (remember that x could be related with other plants different from y in the "attacked by" relation). Having this in mind, there are essentially two ways to approach the usage of RCA within this context, the first one is to model the data into a relational context family, using certain transformations that ensure that there is no semantic loss in the relations [11]. The other one is to extend the input and the algorithm so that it allows the direct representation of these type of relations. In our case, we will go for the second one.

V. *ternary* RCA

As a general notation, we will call the traditional RCA, *binary* RCA, in the sense that its input is a relational context family with binary relationships between objects. Intuitively, the *ternary* RCA extension takes a ternary relational context family, explained below.

A. ternary-Relational context family

Similar to our notation with binary RCA, we call traditional relational context families *binary*-RCF. In contrast, a *ternary*-RCF is a pair (K, R) where K is a set of formal contexts, and R is a set of binary or ternary relations between objects, where at least one of them is ternary. A possible instance could be a (K, R) where $K = \{K_1\}$, $R = \{r\}$, and $r \subseteq K_1 \times K_1 \times K_1$. For this paper, we will consider a ternary relationship between objects of the same table, presented in Table II using the row number of the formal context in Table I.

B. ternary-Relational attributes

In the binary RCA algorithm, for the scaling of each context with a relation r, we add an attribute $\rho\ r : C_i^j$ for each concept in the target lattice $\mathcal{L}_j$, for a given relational scaling operator ρ. When scaling a context K_1 with a relation $r \in K_1 \times K_2 \times K_3$, we add an attribute $\rho\ r_1 : C_{i_1}^2, C_{i_2}^3$ for each combination of concepts in their respective lattices. The subindex in r_1 is to specify the position of the object in the relation, which in the case $K_1 \neq K_2 \neq K_3$ is unnecessary, but in the case the relation repeats contexts, the object could appear in different positions related with the same objects, e.g., (o, x, y) and (x, o, y). Having that said, the attributes $\rho\ r_1 : C_{i_1}^2, C_{i_2}^3$, $\rho\ r_2 : C_{i_1}^2, C_{i_2}^3$, and $\rho\ r_1 : C_{i_2}^3, C_{i_1}^2$ are all different. Considering this, one scaling step on some context $K_u \in K$, and a ternary relation r, adds $t|C^1||C^2|$ new attributes to the context K_u, where t is the amount of times the formal context being scaled is repeated in r.

C. ternary-Relational attributes in PPAf

In this work, we will work with a tiny ternary-relational context family of the PPAf dataset. Particularly, it will consist of only one context K_e (where e stands for example), representing all the possible organisms, plants, or aggressors, and their attributes, which in this simple example will only be if they're used for food or for medical purposes. Additionally, there is a ternary relationship $r_{ps} \subseteq K_e \times K_e \times K_e$ (ps stands for protection system). With this input, in the scaling step, for each concept we add an attribute $\exists_3\ r_{ps,i} : C_j, C_u$ where $1 \leq l \leq 3$ is the position of the object in the relation (because each object could appear in the three positions of the relation), for each combination of concepts in the lattice

TABLE I: Plants, crops and bio-aggressors formal context

	K	Food	Medical
1	Abies sibirica/ Abies/ Pinaceae		
2	Acanthospermum hispidum/ Acanthospermum/ Asteraceae		X
3	Anticarsia gemmatalis/ Anticarsia/ Noctuidae		
4	Allium sativum/ Allium/ Amaryllidaceae	X	X
5	Spodoptera frugiperda/ Spodoptera/ Noctuidae		
6	Spodoptera littoralis/ Spodoptera/ Noctuidae		
7	Spodoptera litura/ Spodoptera/ Noctuidae		
8	CropS/ CropG/ CropF	X	X
9	CropFabaS/ CropFabaG/ Fabaceae	X	
10	Zanthoxylum rhetsa/ Zanthoxylum/ Rutaceae		X
11	Zingiber officinale/ Zingiber/ Zingiberaceae	X	X

Protection	Pest	Crop
1	6	8
2	5	8
4	3	9
10	7	8
11	5	8
11	6	8

TABLE II: Ternary relation

$\mathcal{L}_e$. The ternary scaling algorithm 2 describes how we extend a context according to the ternary version of the operator $\exists_3(o, r, C_1, C_2, i) = \textit{Relations}(o, r, i) \cap C_1.E \times C_2.E \neq \varnothing$ where the *Relations* function is defined in 1.

$$\textit{Relations}(o, r, i) = \begin{cases} \{(x, y) \mid (o, x, y) \in r\} & i = 1 \\ \{(x, y) \mid (x, o, y) \in r\} & i = 2 \\ \{(x, y) \mid (x, y, o) \in r\} & i = 3 \end{cases} \quad (1)$$

D. Useful properties

In this subsection, we will state and prove some useful properties of the $\exists_3$ attributes.

Algorithm 2: Ternary scaling algorithm

Input: $K_i, \mathcal{L}_j, r$, a context, a lattice, and a ternary relation respectively
Output: Scaled K_i

1 $O, A, I \leftarrow K_i$
2 $A^+ \leftarrow A \cup \{\exists\ r_i.\text{name} : C_1, C_2 \mid 1 \leq i \leq 3, (C_1, C_2) \in \mathcal{L}_j \times \mathcal{L}_j\}$
3 $I^+ \leftarrow I \cup \{(o, \exists_3\ r_i.\text{name} : C_1, C_2) \mid o \in O, (C_1, C_2) \in \mathcal{L}_j \times \mathcal{L}_j, 1 \leq i \leq 3, \exists_3(o, r, C_1, C_2, i)\}$
4 **return** O, A^+, I^+

Proposition V.1 (Conservation of position)**.** *Let $C = (X, Y)$ be a formal concept. If Y contains a ternary attribute $\exists_3\ r_l : C_i, C_j$, then all objects in X appear in at least one triple in the position l with at least one object $o_i \in C_i.E$ and one object $o_j \in C_j.E$.*

Proof. Since Y has a ternary attribute of the form $\exists_3\ r_l : C_i, C_j$, we know by definitions of formal concept and the Scaling Algorithm 2 (line 3) that all $o \in X$ attain the condition $\exists_3(o, r, C_i, C_j, l)$. This means that *Relations*$(o, r, l) \cap C_i.E \times C_j.E \neq \varnothing$, or in other words, that all objects in X have at least one pair (x, y) in which they are related to them being in the position l. Thus, all objects in X appear in at least one triple in the position l with at least one object $o_i \in C_i.E$ and one object $o_j \in C_j.E$. □

Proposition V.2 (Conservation of order)**.** *Let $C = (X, Y)$ be a formal concept. If Y contains a ternary attribute $\exists_3\ r_l : C_i, C_j$, then there exists at least one relation in which an object $o_i \in C_i.E$ is at the left of the object $o_j \in C_j.E$, while $o \in X$ is in position l.*

Proof. Since *Relations*$(o, r, l) \cap C_i.E \times C_j.E \neq \varnothing$, let $p \in$ *Relations*$(o, r, l) \cap C_i.E \times C_j.E$. Given the fact that $p = (x, y)$ is an element of $C_i.E \times C_j.E$, x is always at the left of y regardless of the value of l. □

Proposition V.3 (Ternary attributes *non* redundancy)**.** *Let $C = (X, Y)$, C_2, and C_3 be formal concepts. $\exists_3\ r_{ps,1} : C_2, C_3 \in Y$ does not imply $\exists_3\ r_{ps,2} : C_1, C_3 \in C_2.I$ or $\exists_3 r_{ps,3} : C_1, C_2 \in C_3.I$.*

Proof. Let $C = (\{1, 2\}, Y)$, $C_2 = (\{3, 4\}, Y_2)$, $C_3 = (\{5, 6\}, Y_3)$ be three particular formal concepts, if $r = \{(1, 3, 5), (2, 3, 5)\}$, $\exists_3\ r_1 : C_2, C_3 \in Y$, but neither $\exists_3\ r_2 : C_1, C_3 \notin Y_2$ nor $\exists_3\ r_3 : C_1, C_2 \notin Y_3$ since not all the objects in $C_2.E$ nor in $C_3.E$ are related with some object in the respective concepts extents. □

E. Interpretation of the resultant graph

After scaling the context, the final lattice will have ternary edges between the concepts that are part of the protection system. Moreover, thanks to the index in each attribute, the graph maintains the semantic of the relation in the sense that it is possible to tell whether the objects in the context we are studying are aggressors, protected systems, or biopesticides (knowing its index in the relation is enough V.1). On the other hand, we can notice that, even if there seem to be redundancy in the graph, all the $3|C_o|^2$ ternary attributes with their indexes are needed to not lose information V.3, where C_o stands for the set of concepts in the lattice of the formal context I.

Given this situation, the temporal and spacial complexity of the scaling part would be $O(|C_o|^2)$. Since, in order to maintain the semantics of the input relations, it is necessary to ensure that at least we check the incidence of each object in each of the attributes, this algorithm with this input (i.e., only one formal context and one relation) would at least have a temporal complexity of $\Omega(|C_o|^2))$. It is possible, though, to improve in some cases the lower bound of the space complexity by maintaining only the ternary attributes with at least one incidence in I. The ternary attributes with no incidence should be marked and removed, since they add no value to the lattice (they will only be a part of the $\perp$ formal concept extent).

VI. Conclusion and Future Work

In this work, we presented an extension of the widely used RCA framework, or, as we would call it with our notation, binary RCA. We added the possibility of specifying different types of relations, so that no transformation of the input is needed in case it is modelled with ternary relations, unlike in [14] and [21]. On top of that, we presented an initial study about the worst case time complexity bound of the algorithm in terms of the amount of attributes it adds, which is $3|C^1||C^2|$ attributes. This leads us to the questions, "is this approach scalable in the ternary case?", "which optimizations could be applied to the algorithm so that it performs better in the average case?".

To tackle these matters, it would be necessary to benchmark the algorithm in different scenarios and understand where are the points that could be improved. In addition, after the RCA algorithm converges, it is possible that a very dense graph is generated, thus, it is needed for fast algorithms to query it in order to extract information about the relations and also the content of the nodes (formal concepts).

Acknowledgement

This paper is partially supported by funding provided by the STIC AmSud program, Project AGROFAIR, code 21-STIC-01.

References

[1] D. C. Rose, R. Wheeler, M. Winter, M. Lobley, and C.-A. Chivers, "Agriculture 4.0: Making it work for people, production, and the planet," *Land Use Policy*, vol. 100, p. 104933, Jan. 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0264837719319489

[2] M. Lezoche, J. E. Hernandez, M. d. M. E. Alemany Díaz, H. Panetto, and J. Kacprzyk, "Agri-food 4.0: A survey of the supply chains and technologies for the future agriculture," *Computers in Industry*, vol. 117, p. 103187, May 2020. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0166361519307584

[3] H. Panetto, M. Lezoche, J. E. Hernandez Hormazabal, M. del Mar Eva Alemany Diaz, and J. Kacprzyk, "Special issue on Agri-Food 4.0 and digitalization in agriculture supply chains - New directions, challenges and applications," *Computers in Industry*, vol. 116, p. 103188, Apr. 2020. [Online]. Available: https://linkinghub.elsevier.com/retrieve/pii/S0166361519311145

[4] V. Dankevych, Y. Dankevych, N. Bondarchuk, and V. Strilchuk, "The European Green Deal: A Roadmap for Sustainable Nature Management in a Decentralized Environment," *THE PROBLEMS OF ECONOMY*, vol. 1, pp. 185–191, Mar. 2021.

[5] P. J. Silvie, P. Martin, M. Huchard, P. Keip, A. Gutierrez, and S. Sarter, "Prototyping a Knowledge-Based System to Identify Botanical Extracts for Plant Health in Sub-Saharan Africa," *Plants (Basel, Switzerland)*, vol. 10, no. 5, p. 896, Apr. 2021.

[6] J. Savoy and E. Gaussier, "Information Retrieval," in *Handbook of Natural Language Processing, Second Edition*. CRC Press, Taylor and Francis Group, Jan. 2010.

[7] B. Ganter and G. Stumme, *Formal Concept Analysis: Foundations and Applications*. Springer Science & Business Media, Jul. 2005, google-Books-ID: nEh4D4e88NwC.

[8] N. Padhy, "Multi Relational Data Mining Approaches: A Data Mining Technique," *International Journal of Computer Applications*, vol. 57, p. 10, Nov. 2012.

[9] M. Wajnberg, P. Valtchev, M. Lezoche, H. Panetto, and A. Massé, "Mining Process Factor Causality Links with Multi-relational Associations," Marina Del Rey, CA, United States, Sep. 2019, p. 266.

[10] M. Rouane-Hacene, M. Huchard, A. Napoli, and P. Valtchev, "Relational Concept Analysis: Mining Concept Lattices From Multi-Relational Data," *Annals of Mathematics and Artificial Intelligence*, vol. 67, Jan. 2013.

[11] P. Keip, A. Gutierrez, M. Huchard, F. Le Ber, S. Sarter, P. Silvie, and P. Martin, "Effects of Input Data Formalisation in Relational Concept Analysis for a Data Model with a Ternary Relation," in *ICFCA 2019 - 15th International Conference on Formal Concept Analysis*, ser. Lecture Notes in Computer Science, D. Cristea, F. L. Ber, and B. Sertkaya, Eds., vol. 11511. Frankfurt, Germany: Springer International Publishing, Jun. 2019, pp. 191–207. [Online]. Available: https://hal-lirmm.ccsd.cnrs.fr/lirmm-02092148

[12] S. Ferré and P. Cellier, "Graph-FCA in Practice," Jul. 2016, p. 107. [Online]. Available: https://hal.inria.fr/hal-01405491

[13] R. Wille, "The Basic Theorem of triadic concept analysis," *Order*, vol. 12, no. 2, pp. 149–158, Jun. 1995. [Online]. Available: https://doi.org/10.1007/BF01108624

[14] P. Keip, A. Gutierrez, M. Huchard, F. Le Ber, S. Sarter, P. Silvie, and P. Martin, "Effects of Input Data Formalisation in Relational Concept Analysis for a Data Model with a Ternary Relation," in *Formal Concept Analysis*, ser. Lecture Notes in Computer Science, D. Cristea, F. Le Ber, and B. Sertkaya, Eds. Cham: Springer International Publishing, 2019, pp. 191–207.

[15] B. Ganter and R. Wille, *Formal Concept Analysis: Mathematical Foundations*. Springer Berlin Heidelberg, 1999, google-Books-ID: cN1QAAAAMAAJ.

[16] B. Ganter, G. Stumme, and R. Wille, *Formal Concept Analysis: Foundations and Applications*, ser. Lecture Notes in Computer Science. Springer Berlin Heidelberg, 2005. [Online]. Available: https://books.google.fr/books?id=oyb6BwAAQBAJ

[17] R. Agrawal, T. Imieliński, and A. Swami, "Mining association rules between sets of items in large databases," in *Proceedings of the 1993 ACM SIGMOD international conference on Management of data*, ser. SIGMOD '93. New York, NY, USA: Association for Computing Machinery, Jun. 1993, pp. 207–216. [Online]. Available: https://doi.org/10.1145/170035.170072

[18] S. Ferré and O. Ridoux, *A Logical Generalization of Formal Concept Analysis*. Springer Berlin Heidelberg, Mar. 2000, vol. 1867, journal Abbreviation: LNCS Pages: 384 Publication Title: LNCS.

[19] S. Ferré, *Negation, Opposition, and Possibility in Logical Concept Analysis*. Springer Berlin Heidelberg, Jan. 2006, vol. 3874, pages: 145.

[20] M. Wajnberg, "Analyse relationnelle de concepts : une méthode polyvalente pour l'extraction de connaissance," Ph.D. dissertation, Université du Québec à Montréal; Université de Lorraine, CRAN, 2020. [Online]. Available: https://hal.archives-ouvertes.fr/tel-03042085

[21] George Voutsadakis, "Polyadic Concept Analysis," *Order*, vol. 19, no. 3,

pp. 295–304, Sep. 2002. [Online]. Available: https://doi.org/10.1023/A:1021252203599